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MARINE AIRCRAFT EXPERIMENTAL ESTABLISHMENT

FELIXSTOWE

SOME NOTES ON THE CALCULATION OF PRESSURE PICK-UP SENSITIVITY AND THE CONDITIONS FOR MAXIMUM SENSITIVITY

by

J. K. Friswell, B.Sc.

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Report No. F/Res/235

November 1953

MARINE AIRCRAFT EXPERIMENTAL ESTABLISHMENT, FELIXSTOWE, SUFFOLK.

SOME NOTES ON THE CALCULATION OF PRESSURE PICK-UP SENSITIVITY AND THE CONDITIONS FOR MAXIMUM SENSITIVITY

by

J. K. Friswell, B. Sc.

S U M M A R Y

A theoretical analysis is made of the sensitivity of a pressure pick-up of the strain-gauged cantilever type and of the conditions for maximum sensitivity. Two different configurations are treated and the effect of tension in the diaphragm is also considered. An account is given of experiments carried out in order to verify the analysis and to observe the behaviour outside the range of validity of the theory. Suggestions are made for practical pick-up design based on both theory and experiment.

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2. Cantilevers, Strain gage.

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1. INTRODUCTION

Pressure pick-ups are of several different basic designs, intended to operate under various sets of physical conditions. It would therefore be meaningless to compare them merely by their sensitivities. Once a particular type has been chosen, however, it may be necessary to adjust the physical parameters involved for that type in order to obtain maximum sensitivity from it.

This necessity forms the basis for the notes which follow, which refer entirely to one of the basic types of pick-up. This is described in detail later, but consists essentially of a diaphragm to which the pressure to be measured is applied, and a pushrod connecting it to a strain-gauged cantilever. This pick-up is used mainly to measure rapidly varying pressures in a liquid bounding on a solid surface. It will be evident that it only records a mean pressure over the diaphragm, and as it was required to use this type of pick-up to measure pressure in a field with a high space as well as time gradient the diameter of the diaphragm had to be reduced considerably from its usual size. It was at this stage that it became necessary to investigate the conditions for maximum sensitivity at a given diaphragm diameter, since the deflection becomes extremely small at small diameters.

A detailed theoretical analysis of the sensitivity of the pick-up has been made for two different geometrical arrangements and is given below. This analysis is only valid for infinitesimal deflections, and the problem of its extension to larger deflections is a formidable one. The treatment of the large deflection of a diaphragm under either uniform pressure or a centre load is not too difficult, but the results cannot be compounded as the principle of superposition is not valid for these deflections. However, the first order theory will successfully predict the variation of sensitivity with physical parameters at low pressures (which correspond to small deflections) and enable maximum sensitivity to be obtained there. It cannot be expected that the value of any particular parameter for maximum sensitivity will be independent of pressure since, at large deflections, the deflection relations change from linear to cubic form. This being so, it seems reasonable to maximise the sensitivity for low pressures from theory, and to investigate the variations of parameters for maximum sensitivity with increasing pressure. This latter investigation has been carried out for the first pick-up configuration to give guidance on the behaviour of both, and details will be found below.

The effect of initial tension in the diaphragm has also been considered, again only for the first configuration since, as it is impossible to preset a given initial tension, it is only required to know whether it is an advantage to have tension present or not.

Sensitivity is not the only important characteristic of a pick-up. Other factors to be considered are its critical frequencies, natural damping and behaviour under acceleration, which all affect its response. These matters are not directly related to the investigation in hand and have therefore not been dealt with theoretically. Comparisons have, however, been made of several small pick-ups designed for maximum sensitivity with a standard pick-up to determine the changes, if any, in these additional factors.

2. SINGLE CANTILEVER TYPE PICK-UP

2.1. Theoretical treatment for small deflections

The pick-up consists of a circular diaphragm and of a cantilever parallel to it, the two being joined by a pushrod connected to the centre of the diaphragm and the end of the cantilever. On either side of the

/ cantilever

cantilever is fixed an electric bonded wire resistance strain gauge, these two gauges forming two arms of a Wheatstone bridge circuit. The schematic arrangement of the pick-up is illustrated in Figures 1 and 2 (i), AC being the pushrod, AB the cantilever and XY, X' Y' the strain gauges (located arbitrarily on the cantilever for the present). Some simplifications are necessary before the problem can be treated theoretically, and those made mainly relate to the pushrod. This has been assumed to be of negligible cross-section and to be incompressible. This results in the application of a point load to the cantilever, whereas in practice there would also be a moment tending to constrain the cantilever end slope to some extent. To deal with this point realistically the sensitivity has been calculated for two different cases, with the end of the cantilever completely unconstrained in slope and with it constrained to lie parallel to its undeflected position, and the intermediate position briefly considered.

The variables in the problem are (see Figure 2)

h one half of cantilever thickness

x the deflection of the diaphragm and cantilever

q the pressure on the diaphragm

P the thrust in the pushrod

L the cantilever length

a the diaphragm radius

t the diaphragm thickness

other relevant parameters being determined by the choice of materials. Consider first the cantilever AB and assume

(i) pure flexure (i.e. no tension in AB)

(ii) that the weight of AB is negligible in comparison with the shearing forces.

In the deflected positions illustrated in Figure 2 (iii) and (iv), (the two different cases referred to above) the extension of a length element δy between X and Y is $-\frac{h}{R}\delta y$, where R is the local radius of curvature and

$$\frac{1}{R} = \frac{-z_{yy}}{(1+z_y^2)^{3/2}}$$

$$= -z_{yy} \text{ approx if } z_y^2 << 1 , \qquad 2.101$$

z_y, z_{yy} denoting partial derivatives in the usual manner. The total extension of XY is thus

$$\int_X^Y hz_{yy}\, dy = \Big[hz_y\Big]_X^Y = \phi \text{ say} \qquad 2.102$$

and since the extension of X' Y' is $-\phi$, it follows that ϕ is a direct measure of the sensitivity, for a strain gauge of fixed length.

/Initially

Initially we may consider the cantilever to be deflected under an end load P and some bending moment M also acting at the end (Figure 2(v)); the value of M will later be determined by the appropriate end slope conditions. If L is the length of AB and M_Q the bending moment at Q in the sense shown in Figure 2(ii) then

$$M_Q = P(L-y)-M = E_c I z_{yy} \qquad 2.103$$

where E_c is Young's modulus for the cantilever, A its cross-sectional area (= 2bh) and $I = \frac{Ah^2}{3}$.

$$z_{yy} = \frac{P}{E_c I}(L-y) - \frac{M}{E_c I} \qquad 2.104$$

$$\text{and } z_y = \left(\frac{PL-M}{E_c I}\right) y - \frac{Py^2}{2E_c I} \qquad 2.105$$

$$\text{so that } z = \left(\frac{PL-M}{E_c I}\right) \frac{y^2}{2} - \frac{Py^3}{6E_c I} \qquad 2.106$$

$$\text{since } (z)_{y=o} = 0 = (z_y)_{y=0}.$$

$$\text{Now } x = (z)_{y=L} = \frac{PL^3}{3E_c I} - \frac{ML^2}{2E_c I}, \qquad 2.107$$

$$\therefore P = \frac{3E_c Ix}{L^3} + \frac{3M}{2L} = \frac{2E_c bh^3 x}{L^3} + \frac{3M}{2L}. \qquad 2.108$$

The case in which the end of the cantilever is unconstrained in slope gives M = 0, and that in which the end slope is zero $z_y = 0$ at $y = L$ so that $M = \frac{PL}{2}$. The values of ϕ in these two cases, which will be referred to as Cases A and B throughout, are given by

$$\phi = \left[\frac{3xh}{L^3}\left(Ly - \tfrac{1}{2}y^2\right)\right]_X^Y \qquad 2.109\text{ A}$$

$$\text{and } \phi = \left[\frac{6xh}{L^3}\left(Ly - y^2\right)\right]_X^Y \qquad 2.109\text{ B}$$

respectively. More generally, if $z_y = k$ at $y = L$

$$\text{then } M = \frac{PL}{2} - \frac{kE_c I}{L} \qquad 2.107\text{ C}$$

$$\text{and } x = \frac{PL^3}{12E_c I} + \frac{kL}{2} = \frac{PL^3}{8E_c bh^3} + \frac{kL}{2}, \qquad 2.108\text{ C}$$

$$\text{so that } \phi = \left(\frac{6xh}{L^3}\left[Ly - y^2\right]_X^Y + kh\left[\frac{3y^2}{L^2} - \frac{2y}{L}\right]_X^Y\right). \qquad 2.109\text{ C}$$

Case B is obtained directly from Case C by putting k = 0.

/Now

Now consider the diaphragm, which is clamped at its edge. It is subjected to a uniform pressure q and a centre load P, corresponding to the thrust in the pushrod (the moment corresponding to M has been neglected as it would not be expected that a small moment applied about a diameter would affect the centre deflection of the diaphragm). In this section only the case of an initially untensioned diaphragm will be considered, and the centre deflection x_1 under a centre load P is then (Ref. 1)

$$x_1 = \frac{3(1-\sigma^2)\,Pa^2}{4\pi E_d t^3} \qquad 2.110$$

where a is the radius, E_d Young's modulus and σ Poisson's ratio for the diaphragm, and t is its thickness (the diaphragm weight being supposed negligible). [This expression is obtained by writing r = 0 in the full equation of the deflected surface (Ref. 1)

$$w_1 = \frac{Pr^2}{8\pi D} \log \frac{r}{a} + \frac{P}{16\pi D} (a^2 - r^2) \qquad 2.111$$

where w_1 denotes deflection from equilibrium position and $D = \dfrac{E_d t^3}{12(1-\sigma^2)}$ 2.112

so that $x_1 = (w_1)_{r=0}$].

The centre deflection under a uniform pressure is similarly (Ref. 1)

$$x_2 = \frac{3(1-\sigma^2)\,qa^4}{16E_d t^3} \qquad 2.113$$

$$\left[\text{from } w_2 = \frac{q}{64D} (a^2 - r^2)^2 \qquad 2.114\right]$$

so that combining this with 2.110, having due regard to the sense of the deflections,

$$x = \frac{3(1-\sigma^2)\,qa^4}{16E_d t^3} - \frac{3(1-\sigma^2)\,Pa^2}{4\pi E_d t^3} \qquad 2.115$$

$$= \frac{3(1-\sigma^2)\,qa^4}{16E_d t^3} - \frac{3(1-\sigma^2)a^2}{4\pi E_d t^3} \cdot \frac{2E_c bh^3 x}{L^3} \qquad 2.116\text{ A}$$

eliminating P by 2.108 A, when M = 0,

or

$$x = \frac{3(1-\sigma^2)\,qa^4}{16E_d t^3} - \frac{3(1-\sigma^2)a^2}{4\pi E_d t^3} \left\{\frac{8E_c bh^3}{L^3} \left(x - \frac{kL}{2}\right)\right\} \qquad 2.116\text{ B \& C}$$

from 2.108C, for the two cases of non-zero M.

Thus $x = \alpha - \beta x h^3$, say 2.117 A

$$\text{where } \alpha = \frac{3(1-\sigma^2)\,qa^4}{16E_d t^3} \qquad 2.118\text{ A}$$

$$\text{and } \beta = \frac{3(1-\sigma^2)a^2 bE_c}{2\pi E_d t^3 L^3} \qquad 2.119\text{ A}$$

/when

when $M = 0$,

$$\text{or } x = \alpha' - \beta' x h^3 \qquad 2.117 \text{ B \& C}$$

$$\text{where } \alpha' = \alpha + \frac{3k(1-\sigma^2)a^2 E_c b h^3}{\pi E_d t^3 L^2} \qquad 2.118 \text{ B \& C}$$

$$\text{and } \beta' = 4\beta \qquad 2.119 \text{ B \& C}$$

when $M \neq 0$.

$$\text{Thus } x = \alpha\,(1+\beta h^3)^{-1} \quad \text{or} \quad \alpha'(1+\beta' h^3)^{-1} \qquad 2.120$$

as appropriate.

This gives

$$\phi = \left[\frac{3\alpha h\,(Ly-\tfrac{1}{2}y^2)}{L^3\,(1+\beta h^3)}\right]_X^Y \qquad 2.121 \text{ A}$$

$$\text{or } \phi = \left[\frac{6\alpha h\,(Ly-y^2)}{L^3(1+4\beta h^3)}\right]_X^Y \qquad 2.121 \text{ B}$$

$$\text{or } \phi = \left[\frac{6\alpha' h(Ly-y^2)}{L^3(1+4\beta h^3)} + kh\left(\frac{3y^2}{L^2} - \frac{2y}{L}\right)\right]_X^Y \qquad 2.121 \text{ C}$$

These give ϕ in the final form required. It can be seen by inspection at this stage that ϕ will decrease as either t or b increases, in all cases. The value of 'a' will normally be determined by considerations other than that of sensitivity and so only h and L remain as parameters whose effects have still to be determined. Maximum sensitivity under thickness or length variations of the cantilever is now given by

$$\phi_h = 0 \qquad 2.122$$

$$\text{or } \phi_L = 0 \qquad 2.123$$

(suffices again indicating partial derivatives) leading directly to the best values of L and h. Before 2.123 is solved a definite length and position of strain gauge must be selected. If L and h are regarded as simultaneously variable, 2.122 and 2.123 must be solved as simultaneous equations in L and h; otherwise the appropriate one is solved individually.

Case C will not be considered further since normally for these conditions k will be a complicated function of x. Details of the behaviour for the case when k is held constant or is a known function of x can however be calculated from the formulae already given by a procedure exactly analogous to that followed below.

/(a) Suppose

(a) Suppose L fixed and only h variable.

In Case A, $\phi_h = 0$ implies (2.121 A)

$$\left\{\frac{h}{1+\beta h^3}\right\}_h = 0$$

$$\therefore \frac{1-2\beta h^3}{(1+\beta h^3)^2} = 0$$

$\therefore 1 - 2\beta h^3 = 0$ or $1 + \beta h^3 = \infty$, the latter corresponding to minimum sensitivity.

$$\therefore h^3 = \frac{1}{2\beta}$$

$$\text{and } h = \left\{\frac{1}{2\beta}\right\}^{1/3}$$

$$= \left\{\frac{\pi E_d t^3 L^3}{3(1-\sigma^2)a^2 b E_c}\right\}^{1/3}. \qquad 2.124 \text{ A}$$

Similarly Case B gives

$$h = \left\{\frac{\pi E_d t^3 L^3}{12(1-\sigma^2)a^2 b E_c}\right\}^{1/3}. \qquad 2.124 \text{ B}$$

(b) If the strain gauge is supposed to be of length c and to be attached at the root then

$$\phi = \frac{3h\alpha(cL-\frac{1}{2}c^2)}{L^3(1+\beta h^3)} \qquad 2.125 \text{ A}$$

$$\text{or } \phi = \frac{6h\alpha(cL-c^2)}{L^3(1+4\beta h^3)}. \qquad 2.125 \text{ B}$$

Consider first Case A.

$$\phi = \frac{3(cL-\frac{1}{2}c^2)h\eta q a^4}{L^3 t^3 + h^3 \Upsilon a^2 b} \qquad 2.126 \text{ A}$$

(substituting for α and β)

$$\text{where } \eta = \frac{3(1-\sigma^2)}{16E_d} \qquad 2.127$$

$$\text{and } \Upsilon = \frac{3(1-\sigma^2)E_c}{2\pi E_d}. \qquad 2.128$$

Now regarding h and L as simultaneously variable, we require

$$\left.\begin{aligned}\phi_h &= 0\\ \phi_L &= 0\end{aligned}\right\}$$

/But

But $\phi_L = 0$ implies $\dfrac{3ch\eta qa^4}{L^3t^3+h^3\Upsilon a^2b} - \dfrac{9L^2t^3h\eta qa^4\,(cL-\frac{1}{2}c^2)}{(L^3t^3+h^3\Upsilon a^2b)^2} = 0$ 2.129 A

and as $L^3t^3+h^3\Upsilon a^2b = \infty$ corresponds to minimum sensitivity,

$$L^3t^3 - \frac{3}{4}cL^2t^3 - \tfrac{1}{2}h^3\Upsilon a^2b = 0 \qquad 2.130\ A$$

From 2.124 A,

$$\phi_h = 0 \text{ implies } h^3 \doteq \frac{1}{2\beta} = \frac{\pi E_d t^3 L^3}{3(1-\sigma^2)a^2bE_c}$$

$$= \mu^3L^3 \text{ say} \qquad 2.131\ A$$

so that $h = \mu L$.

Substituting in 2.130,

$$L^3(t^3-\tfrac{1}{2}\mu^3\Upsilon a^2b) - \frac{3}{4}cL^2t^3 = 0 \qquad 2.132\ A$$

and as $L \neq 0$ for maximum sensitivity,

$$\underline{L} = \left\{\frac{\frac{3}{4}ct^3}{t^3-\frac{1}{2}\mu^3\Upsilon a^2b}\right\} = \underline{c}, \quad \text{substituting for } \mu \text{ and } \Upsilon$$

$$\text{so that } h = \left\{\frac{1}{2\beta}\right\}^{1/3}_{L=c} \doteq \left\{\frac{\pi E_d t^3c^3}{3(1-\sigma^2)a^2bE_c}\right\}^{1/3} \qquad 2.133\ A$$

This is the result which would have been expected.

For Case B, $\phi = \dfrac{6h\alpha(cL-c^2)}{L^3(1+4\beta h^3)}$

$$= \frac{6(cL-c^2)h\eta qa^4}{L^3t^3+4h^3\Upsilon a^2b}\,. \qquad 2.126\ B$$

$\phi_L = 0$ leads to

$$L^3t^3-\frac{3}{2}cL^2t^3 - 2h^3\Upsilon a^2b = 0 \qquad 2.130\ B$$

and $\phi_h = 0$ (from 2.124 B) to

$$h^3 = \frac{\pi E_d t^3L^3}{12(1-\sigma^2)a^2bE_c}$$

so that from 2.130 B

$$L^3\left(1 - \frac{\Upsilon\pi E_d}{6(1-\sigma^2)E_c}\right) - \frac{3}{2}cL^2 = 0$$

/whence

whence the optimum value of L is

$$L = \frac{3c/2}{1 - \frac{\gamma\pi E_d}{6(1-\sigma^2)E_c}} = 2c, \text{ substituting for } \gamma, \qquad 2.132\text{ B}$$

$$\text{so that } h = \left\{\frac{1}{8\beta}\right\}^{1/3}_{L\,=\,2c} = \left\{\frac{2\pi E_d t^3 c^3}{3(1-\sigma^2)a^2 b E_c}\right\}^{1/3} \qquad 2.133\text{ B}$$

As stated in the introduction, the results obtained in this section are only valid for small deflections, in fact for $x \ll t$. They may be conveniently summarised as under:-

A measure of the pick-up sensitivity is given in the two cases considered in detail by

$$\phi = \left[\frac{3\alpha h(Ly-\frac{1}{2}y^2)}{L^3(1+\beta h^3)}\right]_X^Y \qquad (2.121\text{ A})$$

$$\text{or } \phi = \left[\frac{6\alpha h(Ly-y^2)}{L^3(1+4\beta h^3)}\right]_X^Y \qquad (2.121\text{ B})$$

If only h is regarded as variable, ϕ has a maximum at

$$h = \left\{\frac{1}{2\beta}\right\}^{1/3} = \left\{\frac{\pi E_d t^3 L^3}{3(1-\sigma^2)a^2 b E_c}\right\}^{1/3} \qquad (2.124\text{ A})$$

$$\text{or } h = \left\{\frac{1}{8\beta}\right\}^{1/3} = \left\{\frac{\pi E_d t^3 L^3}{12(1-\sigma^2)a^2 b E_c}\right\}^{1/3} \qquad (2.124\text{ B})$$

while if L and h are both regarded as variable the maximum occurs at

$$\left.\begin{aligned} &L = c \\ &h = \left\{\frac{\pi E_d t^3 c^3}{3(1-\sigma^2)a^2 b E_c}\right\}^{1/3} \end{aligned}\right\} \qquad \begin{aligned}&(2.132\text{ A})\\&(2.133\text{ A})\end{aligned}$$

$$\left.\begin{aligned} &\text{or } L = 2c \\ &h = \left\{\frac{2\pi E_d t^3 c^3}{3(1-\sigma^2)a^2 b E_c}\right\}^{1/3} \end{aligned}\right\} \qquad \begin{aligned}&(2.132\text{ B})\\&(2.133\text{ B})\end{aligned}$$

2.2. The effect of initial tension

It is clear that for small deflections the effect of initial uniform tension T in the diaphragm on the deflections must be to reduce them in the manner

/x_1*

$$x_1^* = x_1\,(1-|f(T)|) \qquad 2.201$$

$$x_2^* = x_2\,(1-|g(T)|) \qquad 2.202$$

where $f(0) = 0 = g(0)$

{or equivalently

$$x_1^* = \frac{x_1}{1+|f_1(T)|} \qquad 2.203$$

$$x_2^* = \frac{x_2}{1+|g_1(T)|} \qquad 2.204$$

where $f_1(0) = 0 = g_1(0)$.}

If in addition the values of T are small these become

$$x_1^* = x_1\,(1-K_1T) \qquad 2.205$$

$$x_2^* = x_2\,(1-K_2T) \qquad 2.206$$

where $K_1 > 0$ and $K_2 > 0$, and $|T| << 1$. Nadai has calculated (Ref.4) that

$$K_2 = \frac{11a^2}{72D} = 0.151\,\frac{a^2}{D} \qquad 2.207$$

$$\text{where } D = \frac{E_dt^3}{12(1-\sigma^2)} \qquad 2.208$$

For dimensional reasons K_1 is also a constant multiple of $\frac{a^2}{D}$, and the value of the constant can be expected to be of the same order as that in 2.207. 2.205 and 6 now enable the effect of initial tension to be examined. Instead of 2.110 and 2.113 we now have

$$x_1^* = \frac{3(1-\sigma^2)Pa^2}{4\pi E_dt^3}\,(1-K_1T) \qquad 2.209$$

$$\text{and } x_2^* = \frac{3(1-\sigma^2)qa^4}{16E_dt^3}\,(1-K_2T) \qquad 2.210$$

$$\therefore\ x = x_2^*-x_1^*$$

$$= \frac{3(1-\sigma^2)qa^4}{16E_dt^3}\,(1-K_2T) - \frac{3(1-\sigma^2)Pa^2}{4\pi E_dt^3}\,(1-K_1T). \qquad 2.211$$

In Case A this gives

$$x = \frac{3(1-\sigma^2)qa^4}{16E_dt^3}\,(1-K_2T) - \frac{3(1-\sigma^2)E_cbh^3xa^2}{2\pi L^3E_dt^3}\,(1-K_1T) \qquad 2.212\ A$$

(from 2.108).

$$= \alpha\,(1-K_2T) - \beta xh^3\,(1-K_1T) \qquad 2.213\ A$$

$$\therefore\ x = \frac{\alpha\,(1-K_2T)}{1+\beta h^3(1-K_1T)} \qquad 2.214\ A$$

$$\text{and } \phi = \frac{3\alpha h(1-K_2T)\,\left[Ly-\tfrac{1}{2}y^2\right]_X^Y}{L^3\left[1+\beta h^3\,(1-K_1T)\right]} \qquad 2.215\ A$$

(from 2.109 A)

/For

For Case B,

$$x = \alpha(1-K_2T) \quad -4\beta(1-K_1T)\ xh^3 \qquad 2.213\ B$$

$$\therefore \quad x = \frac{\alpha(1-K_2T)}{1+4\beta h^3(1-K_1T)} \qquad 2.214\ B$$

$$\text{and } \phi = \frac{6\alpha h(1-K_2T)\left[Ly-y^2\right]_X^Y}{L^3\left[1+4\beta h^3(1-K_1T)\right]} \qquad 2.215\ B$$

The effect of the tension on the optimum cantilever thickness is now given by ϕ_h:-

$$\phi_h = 0 \text{ leads to } h^3 = \frac{1}{2\beta(1-K_1T)} \qquad 2.216\ A$$

$$\text{or } h^3 = \frac{1}{8\beta(1-K_1T)} \qquad 2.216\ B$$

unless $1-K_2T = 0$, which is actually outside the range of validity for T. (The singularity of ϕ at $T = (1 + \frac{1}{\beta h^3})/K_1$ or $(1 + \frac{1}{4\beta h^3})/K_1$ is similarly excluded.) Thus the effect of initial tension is to increase the optimum cantilever thickness in the ratio $\left\{1/(1-K_1T)\right\}^{1/3}$.

Again, the advantage or otherwise of introducing such a tension is determined by ϕ_T:

$$\phi_T = 0 \text{ yields } \frac{(K_1-K_2)\ \beta h^3-K_2}{\left\{1+\beta h^3\ (1-K_1T)\right\}^2} = 0, \qquad 2.217\ A$$

which has only a solution at $T = \infty$, which lies outside the range of T for which the analysis is valid. This indicates that ϕ is monotonic in T as T increases from zero, whence the sign of ϕ_T is always that of $(\phi_T)_{T=0}$. From 2.217 A, $(\phi_T)_{T=0} \gtrless 0$ according as

$$\beta h^3 \gtrless \frac{K_2}{K_1-K_2} \qquad 2.218\ A$$

This is therefore the determining condition for the introduction of a small tension in Case A, and a similar analysis of Case B gives a condition

$$\beta h^3 \gtrless \frac{K_2}{4(K_1-K_2)} \qquad 2.218\ B$$

/2.3. Use

2.3. Use of diaphragm with unclamped edge

If the edge of the diaphragm is freely supported, the deflection relations become (Ref. 1)

$$x_1 = \frac{(3+\sigma)\ Pa^2}{16\pi(1+\sigma)D} \qquad 2.301$$

and $$x_2 = \frac{(5+\sigma)\ qa^4}{64(1+\sigma)D} \qquad 2.302$$

so that $x = x_2 - x_1$

$$= \frac{(5+\sigma)\ qa^4}{64(1+\sigma)D} - \frac{(3+\sigma)\ Pa^2}{16\pi(1+\sigma)D} \qquad 2.303$$

which in Case A becomes

$$x = \frac{(5+\sigma)qa^4}{64(1+\sigma)D} - \frac{(3+\sigma)a^2}{16\pi D(1+\sigma)} \cdot \frac{2E_c bh^3 x}{L^3}$$

$$= \alpha_1 - \beta_1\ xh^3 \qquad 2.304\ A$$

where $$\alpha_1 = \frac{(5+\sigma)\alpha}{1+\sigma} \qquad 2.305$$

and $$\beta_1 = \frac{(3+\sigma)\beta}{1+\sigma} \qquad 2.306$$

whence $$\phi = \left[\frac{3\alpha_1 h(Ly-\frac{1}{2}y^2)}{L^3(1+\beta_1 h^3)}\right]_X^Y \qquad 2.307\ A$$

while for Case B,

$$\phi = \left[\frac{6\alpha_1 h(Ly-y^2)}{L^3(1+4\beta_1 h^3)}\right]_X^Y \ . \qquad 2.307\ B$$

The use of this method of diaphragm attachment will therefore be an advantage if

$$\frac{\alpha_1}{1+\beta_1 h^3} > \frac{\alpha}{1+\beta h^3}\ (A) \text{ or } \frac{\alpha_1}{1+4\beta_1 h^3} > \frac{\alpha}{1+4\beta h^3}\ (B) \qquad 2.308$$

$$\text{i.e } h^3 > \frac{\alpha-\alpha_1}{\alpha_1\beta-\alpha\beta_1} = \frac{-2}{\beta}\ (A) \text{ or } h^3 > -\frac{1}{2\beta}\ (B) \qquad 2.309$$

which means in all practical cases, the sensitivity being increased by a multiplicative factor

$$(5+\sigma)\left\{\frac{1+\beta h^3}{1+\sigma+\beta h^3(3+\sigma)}\right\}\ (A) \text{ or } (5+\sigma)\left\{\frac{1+4\beta h^3}{1+\sigma+4\beta h^3(3+\sigma)}\right\}\ (B).$$

/2.4. Correlation

2.4. Correlation with experiment

The various results derived in previous sections relate only to deflections which are small compared with the diaphragm thickness. As the deflections involved in practice considerably exceed this limit, the correlation of the results with experiment falls naturally into two parts, the confirmation of results predicted for small deflections and observation of the behaviour for large deflections.

The pick-up which was in use before the commencement of this investigation contained a German silver diaphragm and beryllium copper cantilever, the values of the various parameters being as under:-

$L = 1$ in.

$h = 0.1$ in.

b mean $= 0.3$ in. (slightly tapered).*

$t = 0.004$ in.

$a = 0.5$ in.

$\sigma = 0.37$

$E_d = 16.8 \times 10^6$ lb./in.^{2}

$E_c = 18.5 \times 10^6$ lb./in.^{2}

$c = 0.5$ in.

This pick-up has been used as a standard for comparison purposes, and where not otherwise mentioned the values of the parameters have been maintained at these values in the tests described.

Owing to its method of fitting, the diaphragm in this pick-up was subjected to initial tension, approximately isotropic and homogenous. This tension varied from diaphragm to diaphragm and the first test carried out was to obtain deflection curves under both uniform pressure and a centre load for two such diaphragms and also for one so mounted that it had no initial tension. These curves are given in Figures 3 and 4, Diaphragm C being that with no initial tension.

These curves illustrate Sections 2.1 and 2.2. above. The relations 2.110 and 2.113 are the theoretical equations of the deflection curves for Diaphragm C. They are, however, only valid as long as the middle surface of the diaphragm remains the neutral surface. This will only be the case when x is small compared with t, so that the two equations should give the tangents to the two experimental curves at the origin. Inserting the numerical values given above the equations become

$$x = 0.0094\, q \qquad 2.401$$

and $$x = 0.0479\, P \qquad 2.402$$

respectively.

It will be seen from Figures 3 and 4 that these lines are in fact the required tangents, the rate of fall-away of the experimental curves with increasing load being greater in the case of the centre load.

/It

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* The taper is not significant since sensitivity for a parallel sided cantilever only varies slightly with b over the range considered (see Figure 9).

[It may be noted that the equations of the curves for Diaphragm C for large deflections can actually be calculated. 2.113 becomes approximately

$$\frac{x_2}{t} + 0.488\left(\frac{x_2}{t}\right)^3 = \frac{3}{16}\cdot\frac{q}{E_d}\left(\frac{a}{t}\right)^4 (1-\sigma^2) \qquad 2.403$$

(cf. the various formulae quoted in Ref. 2). Similar alterations would have to be made to all the relations quoted in the previous sections under these conditions, including the value of ϕ, and the deflection relations calculated by some method not involving the superposition principle. The requisite calculations were not felt to be justified for the purpose in hand as, in any case, it would not be expected that it would be possible to maximise sensitivity simultaneously for small and large deflections.]

In Figures 5 and 6 are given the sensitivity curves obtained by fitting one cantilever and strain gauge system to Diaphragms B and C in turn; meter readings on these and all other figures refer to the output meter reading of the amplifying circuit used, conditions in this circuit being the same for all experiments. The cantilever system was exactly the same as that already described except that only one strain gauge, on the lower face, was used, the circuit being completed by an unstrained gauge separately mounted. The cantilever thickness was reduced by grinding in steps from 0.090 in. to 0.010 in. Curves have been obtained at various pressures, the small ones corresponding to deflections coming within the theoretical treatment and the large ones to those lying outside it. That the curves have had to be presented in two sections is due to an enforced change of strain gauge; as it is never possible to mount two strain gauges in exactly the same position, the characteristics for the two halves differ slightly, but this will not affect the general variation.* Additional curves have, however, been included showing the variation of sensitivity with cantilever thickness over a smaller range for two pick-ups, one with a pretensioned and one with an untensioned diaphragm (Figures 7 and 8): these do not represent the same cantilever systems and so are not directly comparable, but they do enable the variation of peak sensitivity to be studied more accurately than in Figures 5 and 6. The value of b for these two cantilevers was constant at 0.25 in. as against a mean of 0.30 in. for the previous case. These cases therefore give a slightly better approximation to the theoretical case.

Substitution in 2.124 A and B shows that maximum sensitivity is predicted to occur at $2h = 0.0196$ in. when $b = 0.30$ in. and at $2h = 0.0208$ in. when $b = 0.25$ in. for Case A, and at $2h = 0.0123$ in. and 0.0131 in. for Case B. It will be seen from a comparative study of Figures 5-8 that the peak sensitivities do in fact occur between the predicted thicknesses for the two extreme cases (allowing some little latitude, both for experimental error in measurements and for approximations made in the theory). Case A is a better guide to actual conditions than Case B, but the relative merits of the two cases will in general depend on the dimensions of the pushrod and other components used; the pushrod in the cases illustrated was of circular cross-section, 0.1 in. diameter and 0.6 in. long. There is surprisingly little variation of the peak position with increasing pressure; the very slight apparent increase in the thickness for maximum sensitivity with increase of pressure may or may not be actually present. The curves would seem to indicate that the theoretical treatment of Section 2.1 is valid in $x < t$ rather than $x \ll t$. (Maximum deflections obtained were of the order of 0.01 in. for $q = 3$).

Figures 5, 6 and 8 also illustrate Section 2.2. In neither case does there appear to be any significant change in peak position at low pressures due to the presence of initial tension in the diaphragm, but in the case of the diaphragm used to obtain Figures 5 and 6 there is a small

/shift

* For the same reason, Figures 7 and 8 should not be compared quantitatively with Figures 5 and 6.

shift upward with increasing pressure in the value of optimum cantilever thickness. This effect is still however not of a size which would be significant in practice. That the effect is not apparent in Figure 8 may be due to the difference in diaphragm tension between the two cases or to the differences in the values of b.

Comparison of the two sets of curves given in Figures 5 and 6 shows the effect of introducing tension into the diaphragm of a pick-up of which the characteristics are already known. Sensitivity is in general reduced in the range of cantilever thicknesses considered, at least for small pressures (corresponding to small deflections). At large pressures, there is little difference between the two cases. Calculations based on Figure 3 indicate that the value of T for this diaphragm is too large for the relations of Section 2.2 to be valid, so that this case involves one approximation additional to the previous one - a term in T^2 can be expected to occur in the more exact relations relevant to this case. The results are however a good illustration of Section 2.2 and should not give results markedly different (qualitatively) from those for small tensions; it might, however, be misleading to compare these results with the condition 2.218 for the introduction of small tensions, though the general conclusion that a critical value of cantilever thickness exists above which a tensioned diaphragm is an advantage seems to be confirmed by the rapid convergence of the sensitivity curves of Figure 5 in the region of 2h = 0.1 in.

For comparison with the above results a graph of the theoretical sensitivity curves based on 2.125 A is given in Figure 9 for three cantilever breadths, and a pressure of 3 lb./sq. in.

The results of the experiments were taken to indicate that little loss in sensitivity would occur in practical pick-up design if the optimum physical dimensions of the cantilever were calculated from the small deflection relations for the most appropriate cantilever end condition and that the introduction of a tensioned diaphragm, while reducing the sensitivity, would not have a large effect on the optimum dimensions if the tension were kept reasonably small. Figure 10 gives calibration curves of a pick-up designed on these principles and of a pick-up of the "standard" type already mentioned. This shows some improvement, though, owing to differences in strain gauge mountings, etc., this is not necessarily the maximum improvement which can be achieved. The variation of sensitivity with diaphragm radius, observing optimum cantilever thicknesses for Case A is illustrated in Figure 11.

As the results above were considered generally satisfactory and in accord with the small deflection theory, the calculations for the other configuration have been carried out for small deflections, no initial tension and with a clamped edge diaphragm, no further experiments having been performed to verify these calculations.

3. TWIN CANTILEVER TYPE PICK-UP

3.1. Theoretical treatment for small deflections

This pick-up configuration was designed to give smaller overall dimensions than the previous one. It employs twin cantilevers set back to back, the pushrod now dividing as shown in Figures 12 and 13 to deflect both cantilevers. Only two strain gauges are used, each covering one side of both cantilevers, so that there is now an unused portion of each where it covers the cantilever root.

Most of the treatment is identical with that of the previous case, only slight alterations in notation being necessary. The length of the common encastered root is denoted by r and the individual lengths of the cantilevers by L_1. The "exposed" length of strain gauge on each cantilever is s, so that the total length of each gauge is 2s + r.

/Other

Other notation remains as before (see Figure 13). Only Case A has been treated as Case B is unlikely to be realised with this arrangement but the necessary alterations can be carried out as before.

If the thrust in the pushrod remains P at the diaphragm, the end thrust on each cantilever will be P/2, so that adapting 2.108 A we have (for AB say)

$$\frac{P}{2} = \frac{3E_cIx}{L_1^3} = \frac{2E_cbh^3x}{L_1^3} \qquad 3.101$$

A measure of the sensitivity of this pick-up arrangement is given by

$$\phi_1 = \Big[hz_y\Big]_B^X \qquad 3.102$$

(cf. 2.102).

This is not the same function of the absolute sensitivity as ϕ in the previous section, but it is still a measure of the sensitivity of the whole pick-up.

By the same reasoning as was employed before (cf. 2.109 A)

$$\phi_1 = \left[\frac{Ph}{2E_cI}\;(L_1y-\tfrac{1}{2}y^2)\right]_B^X = \left[\frac{3xh}{L_1^3}\;(L_1y-\tfrac{1}{2}y^2)\right]_B^X$$

$$= \frac{3xh}{L_1^3}\;(sL_1-\tfrac{1}{2}s^2) \qquad 3.103$$

The diaphragm deflection relation remains, as in 2.115,

$$x = \frac{3(1-\sigma^2)qa^4}{16E_dt^3} - \frac{3(1-\sigma^2)Pa^2}{4\pi E_dt^3} \qquad 3.104$$

but, because of the difference between 3.101 and 2.108 A, reduces to

$$x = \alpha - 2\beta xh^3 \qquad 3.105$$

so that

$$x = \alpha\;(1+2\beta h^3)^{-1} \qquad 3.106$$

and

$$\phi_1 = \frac{3\alpha h\;(sL_1-\tfrac{1}{2}s^2)}{L_1^3\;(1+2\beta h^3)} \qquad 3.107$$

It is evident that if $(\phi_1)_{L_1} = 0$ is solved then optimum sensitivity will occur at a value of L_1 equal to the exposed length of the strain gauge on one cantilever, i.e. $L_1 = s$. 3.108

/The

The calculation of this will not be carried out. Similarly $(\phi_1)_h = 0$ will lead to an optimum beam thickness 2h where

$$h = \left\{\frac{1}{4\beta}\right\}^{1/3} = \left\{\frac{\pi E_d t^3 L_1^3}{6(1-\sigma^2)a^2 b E_c}\right\}^{1/3} \qquad 3.109$$

Simultaneous solution will give

$$\begin{cases} L_1 = s & (3.108) \\ h = \left\{\dfrac{\pi E_d t^3 s^3}{6(1-\sigma^2)a^2 b E_c}\right\}^{1/3} & 3.110 \end{cases}$$

It should be remembered that if the total length of a strain gauge is c, then $s = \frac{1}{2}(c-r)$. 3.111

A comparison of this and the previous configuration may now be made. It is necessary to take a gauge of fixed length c for the two cases and compare ϕ with $2\phi_1$. Supposing the gauge in the first case to be attached at the root, 2.121 A gives

$$\phi = \frac{3ah(cL-\frac{1}{2}c^2)}{L^3(1+\beta h^3)}$$

while from 3.107

$$2\phi_1 = \frac{6ah(sL_1-\frac{1}{2}s^2)}{L_1^3(1+2\beta h^3)}$$

The only fair comparison is the ideal case $r = 0$, $L = 2L_1$, all other parameters being the same in both cases, when we have $s = \frac{1}{2}c$ so that

$$\phi = \frac{3ah(cL-\frac{1}{2}c^2)}{L^3(1+\beta h^3)} \qquad 3.112$$

$$\text{and } 2\phi_1 = \frac{12ah(cL-\frac{1}{2}c^2)}{L^3(1+2\beta h^3)} \qquad 3.113$$

$$= 4\phi\left(\frac{1+\beta h^3}{1+2\beta h^3}\right) \qquad 3.114$$

$$> \phi \text{ for all positive values of } \beta h^3,$$

and the ratio of sensitivities is

$$\frac{2\phi_1}{\phi} = \frac{4(1+\beta h^3)}{1+2\beta h^3} \qquad 3.115$$

which is quite high.

The more practical case is that in which $r \neq 0$, and if then $2L_1 + r = L$ and $s = \frac{1}{2}(c-r)$ then

$$2\phi_1 = \frac{12ah\left\{(c-r)(L-r) - \frac{1}{2}(c-r)^2\right\}}{(L-r)^3(1+2\beta h^3)} \qquad 3.116$$

/and

$$\text{and } \left(\frac{2\phi_1}{\phi}\right) = \frac{4(1+\beta h^3)}{(1+2\beta h^3)} \left\{\frac{L}{L-r}\right\}^3 \left\{\frac{(c-r)(L-r) - \frac{1}{2}(c-r)^2}{cL-\frac{1}{2}c^2}\right\}.$$

$$\text{i.e } \frac{4(1+\beta h^3)}{(1+2\beta h^3)} \left\{\frac{L}{L-r}\right\}^3 \left\{\frac{(cL-\frac{1}{2}c^2) - rL-\frac{1}{2}r^2}{(cL-\frac{1}{2}c^2)}\right\}$$

which reduces the advantage somewhat.

3.2. Correlation with experiment

As already stated, no additional experiments were performed to check the results of this section. A graph is, however, included (Figure 14) to show the relative sensitivities of the two types of pick-up, with the same size of diaphragm and identical strain gauges, both being designed to optimum dimensions from Sections 2.1 and 3.1. (The relevant values of parameters additional to those already given are $L_1 = 0.3$ in., $r = 0.19$ in., giving optimum thickness 0.008 in. for twin cantilever type pick-up). This confirms that the twin cantilever type is the more sensitive, as predicted in Section 3.1. That the effect of introducing tension in the diaphragm is similar to the previous case is indicated in Figure 15, where calibration curves are given for pick-ups with normal* tension and with very low tension in the diaphragm respectively.

4. SECONDARY CHARACTERISTICS

In order to test the secondary characteristics of pick-ups referred to in the introduction, three single cantilever type pick-ups were constructed with diaphragm radii 0.5, 0.25 and 0.125 in. respectively and with cantilever thicknesses determined from 2.124 (Case A) and one twin cantilever type of diaphragm radius 0.5 in. and with cantilever thickness determined from 3.109. These were compared with a standard pick-up for behaviour under acceleration and for natural frequency.

In no case was there a deflection equivalent to a uniform pressure of more than 0.01 lb./sq.in. for an acceleration of g ft./sec.^{2}. This figure was obtained with the twin cantilever type, the standard pick-up having a deflection equivalent to 0.005 lb./sq.in. and the others deflections which were not measurable. All the pick-ups were therefore considered satisfactory in this respect.

To measure the natural frequency of the pick-ups, each was given an initial deflection and then released from rest. In this manner, what may be termed the fundamental mode would be excited (i.e. the combination of the lowest modes of both diaphragm and cantilever). Photographic records, however, revealed no trace whatever of any subsequent oscillations in any of the cases, and it can therefore be concluded that the fundamental frequencies were considerably in excess of the limiting frequency which could be detected by the recording system (c. 60 cycles/sec.). Since the recording system was that which is normally used in conjunction with such pick-ups, it was therefore concluded that the natural frequencies would not create any practical difficulties.

5. CONCLUSIONS

Taking both the theoretical and experimental results into consideration, it can be said that, if it is required to design a pick-up of either of the types considered so as to obtain maximum sensitivity from it, then the optimum cantilever thickness can be obtained from relations 2.124 or 3.109, either Case A or Case B being chosen to accord with the cantilever end conditions most nearly obtaining with the system used; if there is doubt on this point the mean thickness for the two

/cases

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* i.e. tension of the same order as that in the standard single-cantilever type pick-up.

cases can be used without a serious loss of sensitivity, though Case A is more likely to be realised than Case B. The breadth of the cantilever and thickness of the diaphragm should be kept as small as possible, and the length of the cantilever as nearly as possible equal to the length of the strain gauge element (Case A) or twice this length (Case B). The presence of initial tension in the diaphragm will not appreciably affect the optimum dimensions, but will reduce the sensitivity and the tension should therefore be kept as small as possible, bearing other requirements in mind. Even greater sensitivity can be obtained by using a diaphragm which is effectively freely supported, but this is not likely to be a practical proposition.

No significant change in the natural frequencies of the pick-up or in its behaviour under acceleration need be expected to arise from modifications to give optimum dimensions.

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/LIST OF SYMBOLS

LIST OF SYMBOLS

a	diaphragm radius
b	cantilever width
c	length of strain gauge
h	one half of cantilever thickness
k	cantilever end slope
q	pressure on diaphragm
r	radial co-ordinate for diaphragm (Section 2) or length of encastered root of strain gauge (Section 3).
s	"exposed" length of strain gauge on each of twin cantilevers
t	diaphragm thickness
w_1	deflection of any point of untensioned diaphragm under centre load
w_2	deflection of any point of untensioned diaphragm under uniform pressure
x	deflection of end of cantilever
x_1	deflection of centre of untensioned diaphragm under centre load
x_2	deflection of centre of untensioned diaphragm under uniform pressure
x_1^*	centre deflection of pretensioned diaphragm under centre load
x_2^*	centre deflection of pretensioned diaphragm under uniform pressure
y, z	co-ordinates relative to cantilever base (see Figure 2)
A	cross-sectional area of cantilever = $2bh$
D	$= \dfrac{E_d t^3}{12(1-\sigma^2)}$
E_c	Young's modulus of cantilever
E_d	Young's modulus of diaphragm
I	$= Ah^2/3$
K_1, K_2	tension correction constants (see 2.205 and 6)
L	cantilever length (single cantilever type pick-up)
L_1	cantilever length (twin cantilever type pick-up)
M	bending moment in cantilever
P	force in pushrod
R	radius of curvature of cantilever
T	tension in diaphragm

/(A,B,

(A,B,C,D,Q,X,Y,X',Y' also used geometrically as shown in Figures 2 and 13).

$$\alpha = \frac{3(1-\sigma^2)qa^4}{16E_d t^3}$$

$$\alpha' = \alpha + \frac{3k(1-\sigma^2)a^2E_c bh^3}{\pi E_d t^3 L^2}$$

$$\alpha_1 = \frac{(5+\sigma)\alpha}{1+\sigma}$$

$$\beta = \frac{3(1-\sigma^2)a^2bE_c}{2\pi E_d t^3 L^3}$$

$$\beta' = 4\beta$$

$$\beta_1 = \frac{(3+\sigma)\beta}{1+\sigma}$$

$$\gamma = \frac{3(1-\sigma^2)E_c}{2\pi E_d}$$

ϕ sensitivity of single cantilever type pick-up (see 2.102).

ϕ_1 sensitivity of twin cantilever type pick-up (see 3.102).

σ Poisson's ratio for diaphragm

$$\eta = \frac{3(1-\sigma^2)}{16E_d}$$

$$\mu = \left\{\frac{\pi E_d t^3}{3(1-\sigma^2)a^2bE_c}\right\}^{1/3}$$

Letters used as suffices denote partial derivatives.

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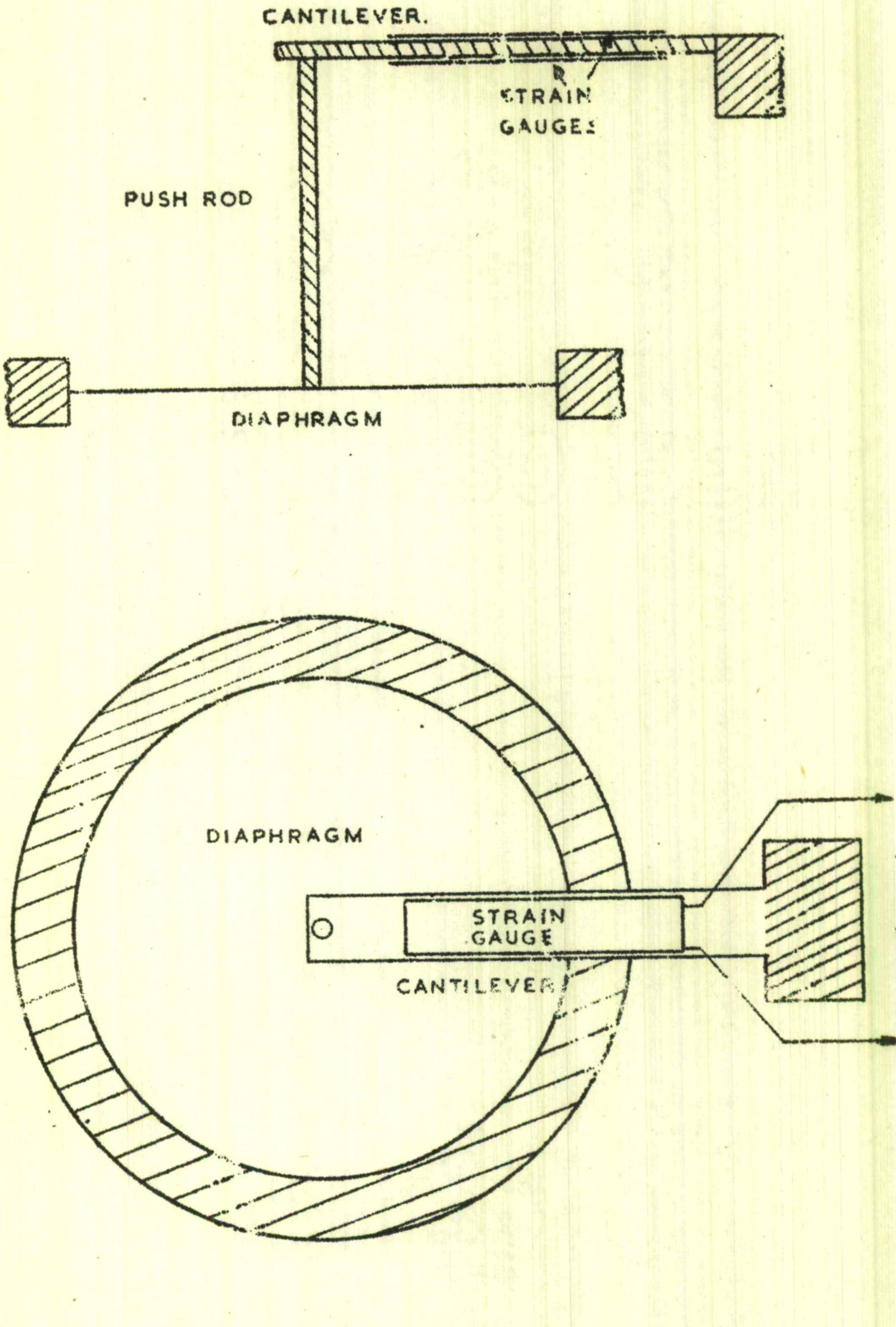
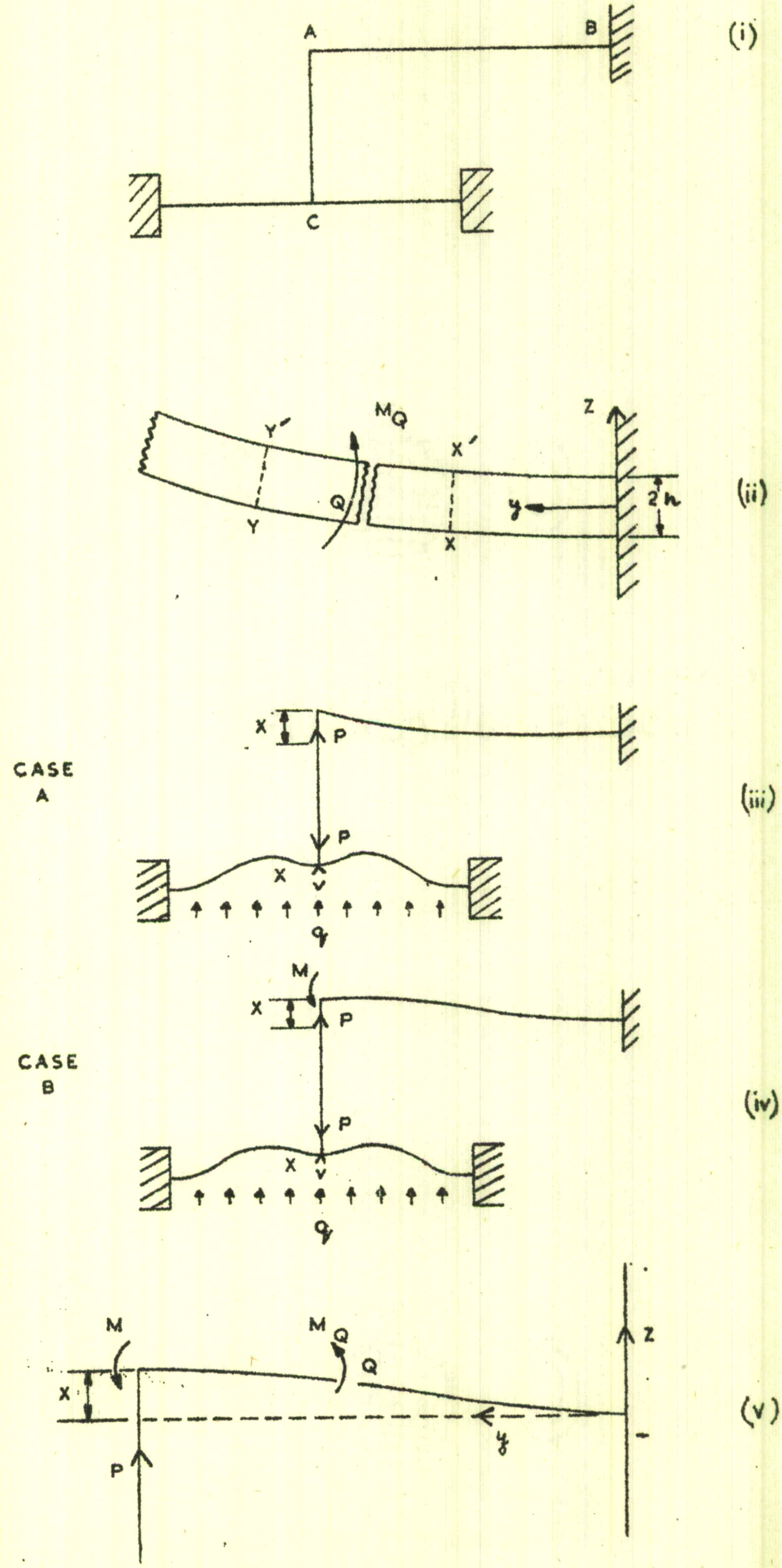
FIG. 1.



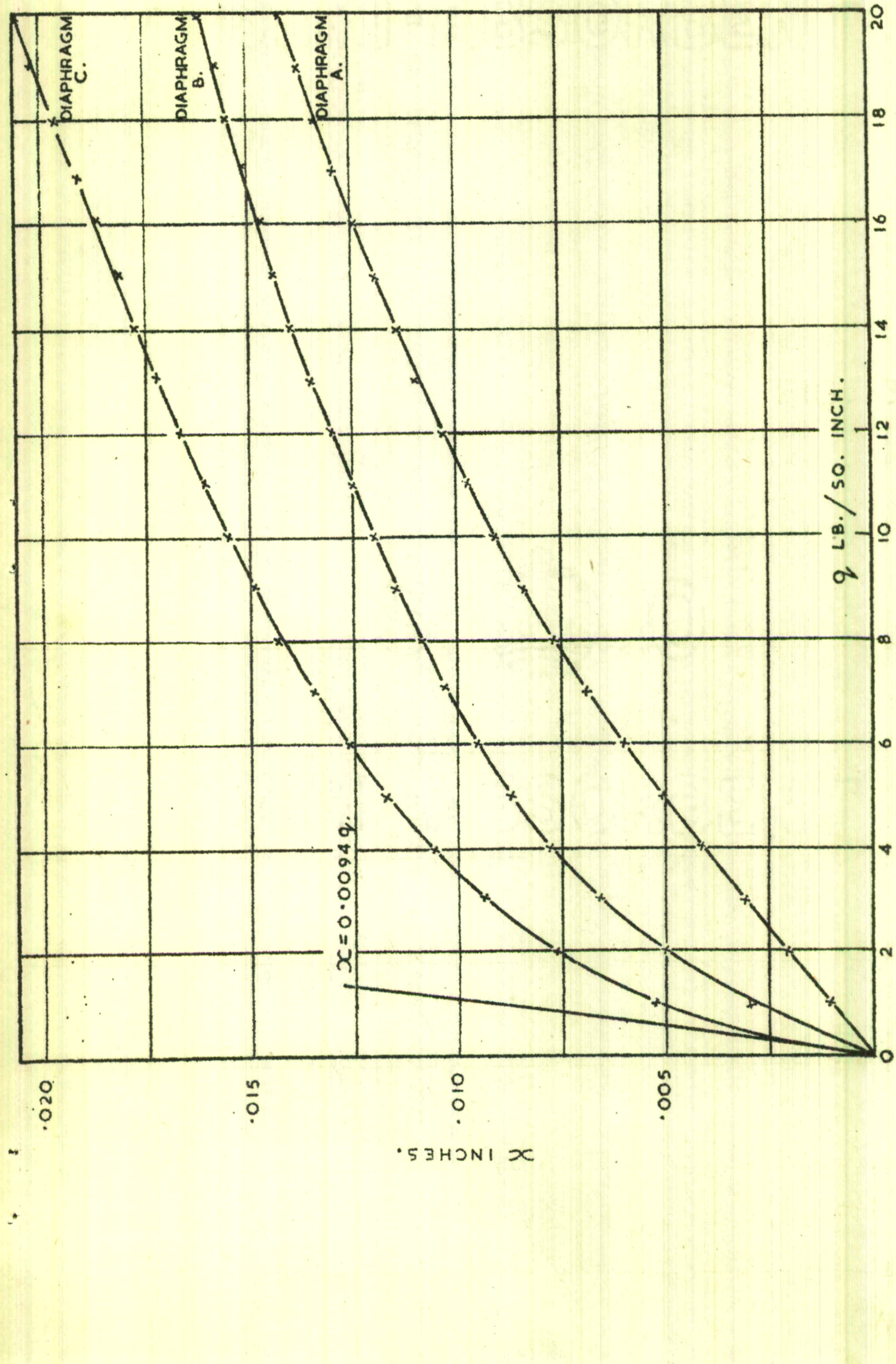
SCHEMATIC ARRANGEMENT OF SINGLE CANTILEVER TYPE PICKUP.

FIG. 2.



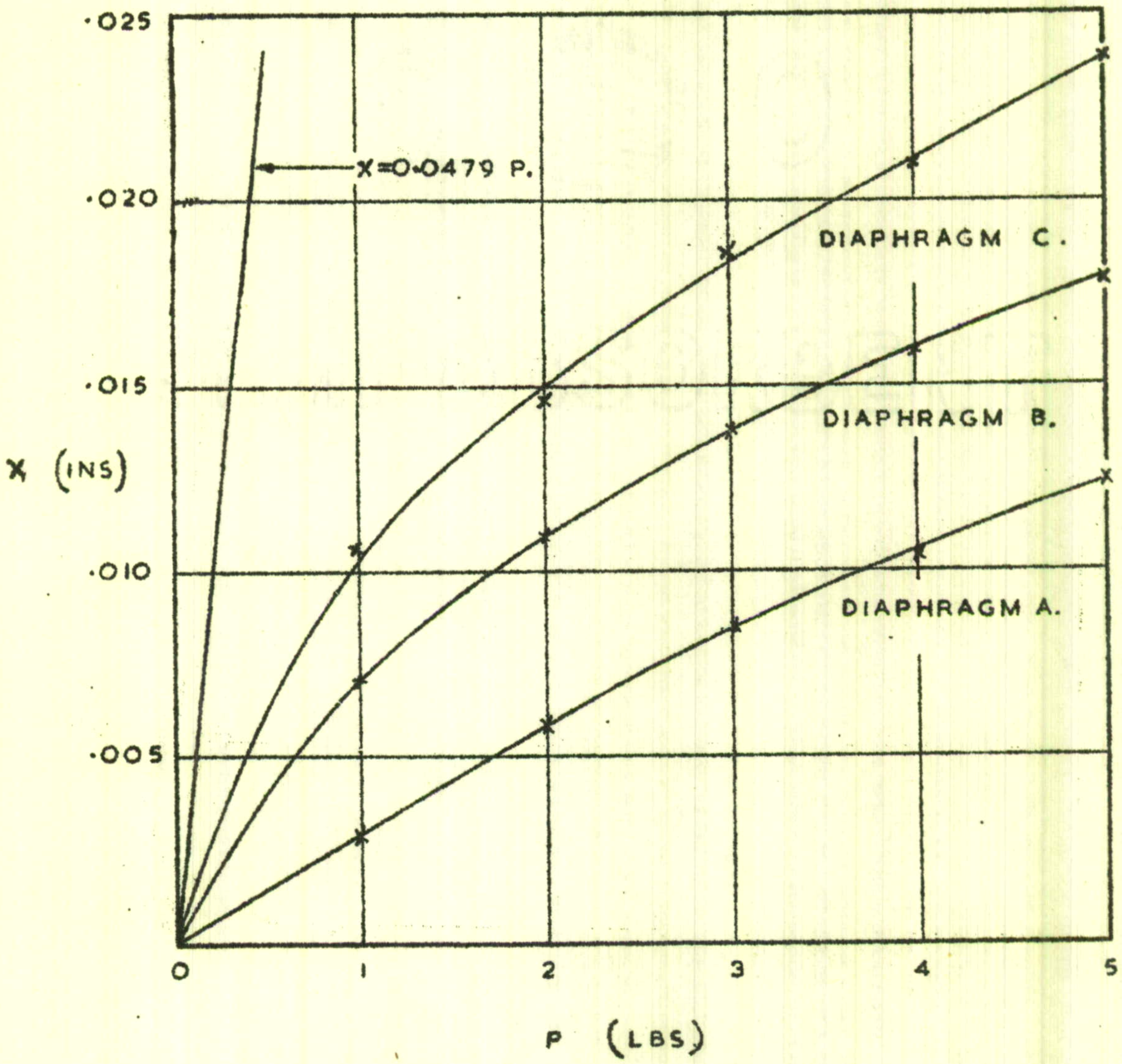
NOTATION USED IN SECTION 2.

FIG. 3.

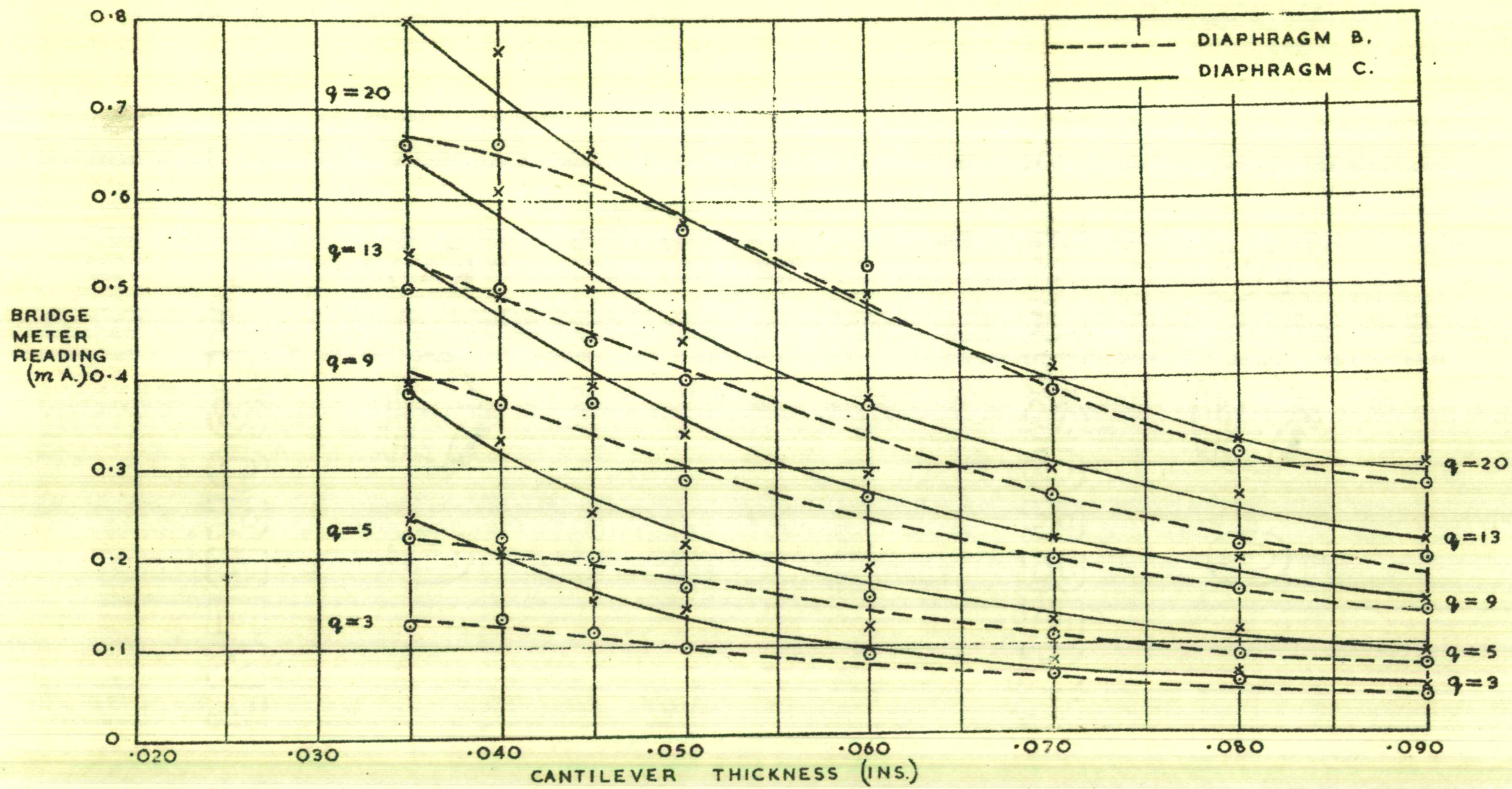


DIAPHRAGM DEFLECTION CURVES UNDER UNIFORM PRESSURE.

FIG. 4.



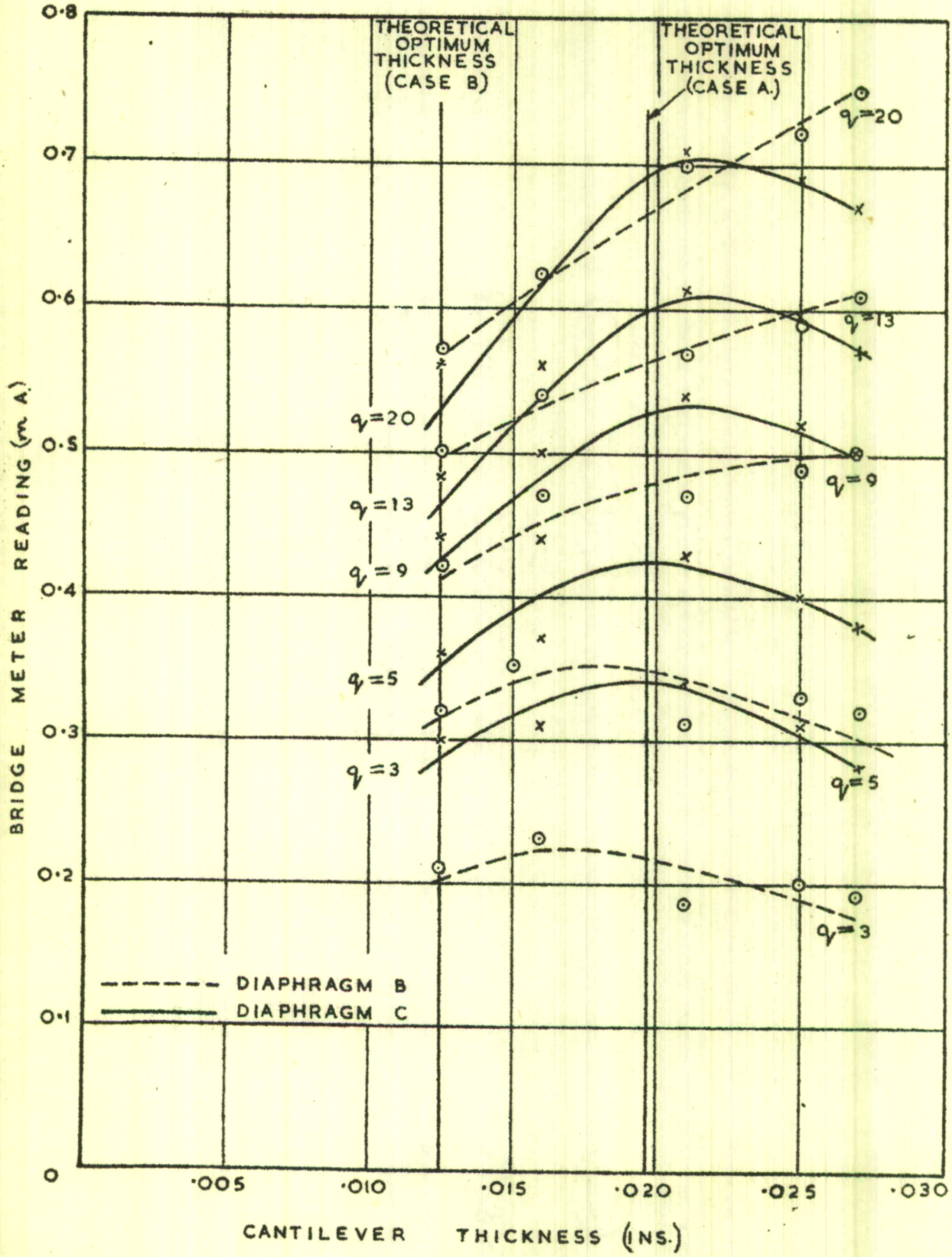
DIAPHRAGM DEFLECTION CURVES
UNDER CENTRE LOADS.



EFFECT OF CANTILEVER THICKNESS ON PICKUP SENSITIVITY, WITH AND WITHOUT INITIAL DIAPHRAGM TENSION, PART I (SINGLE CANTILEVER TYPE PICKUP)

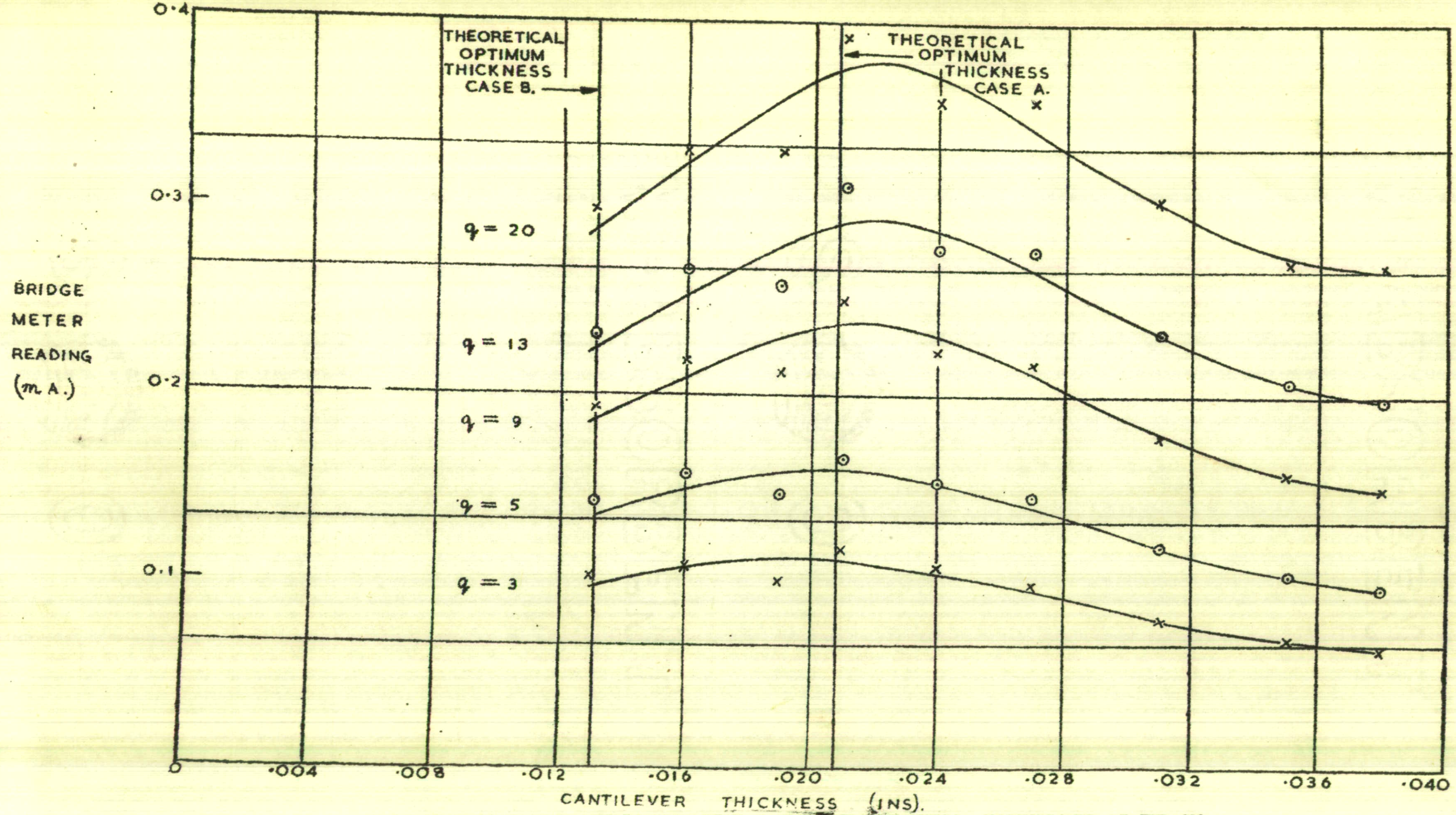
FIG. 5.

FIG. 6.



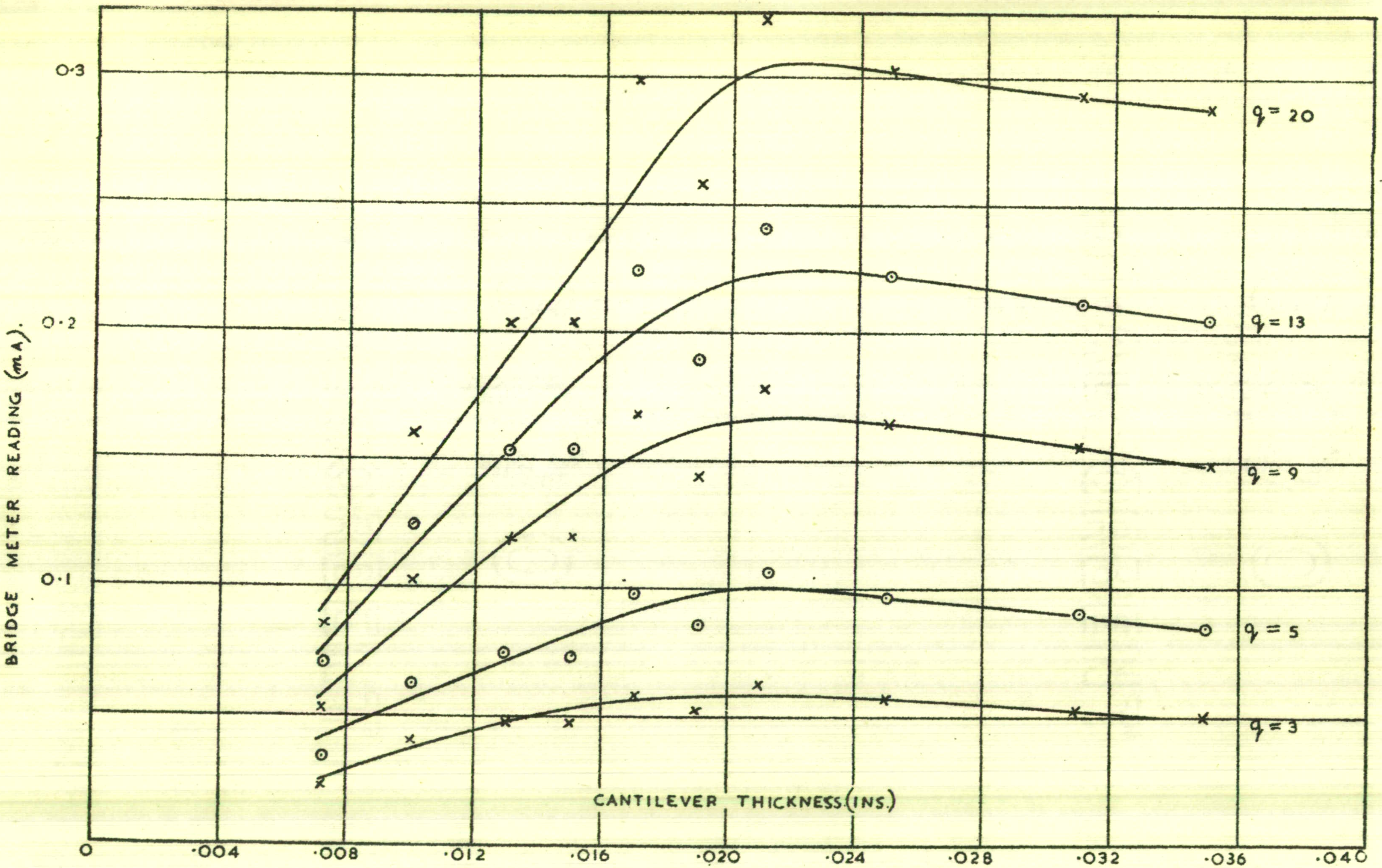
EFFECT OF CANTILEVER THICKNESS ON PICKUP SENSITIVITY, WITH AND WITHOUT INITIAL DIAPHRAGM TENSION, PART II

(SINGLE CANTILEVER TYPE PICKUP.)



VARIATION OF PICKUP SENSITIVITY WITH CANTILEVER THICKNESS, UNTENSIONED DIAPHRAGM.
(SINGLE CANTILEVER TYPE PICKUP).

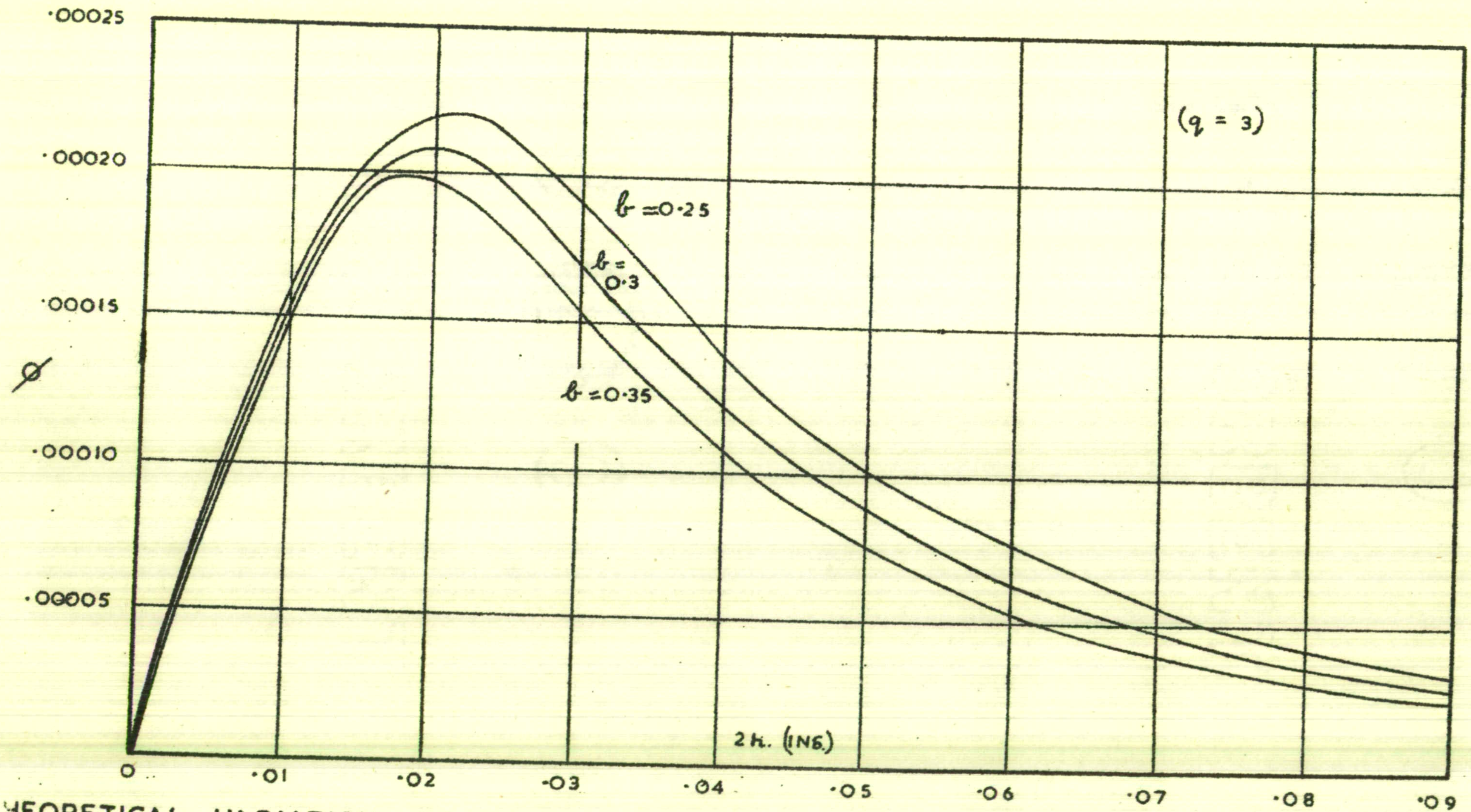
FIG. 7.



VARIATION OF PICKUP SENSITIVITY WITH CANTILEVER THICKNESS, PRETENSIONED DIAPHRAGM (SINGLE CANTILEVER TYPE PICKUP)

FIG. 8.

FIG. 9.



THEORETICAL VARIATION OF PICKUP SENSITIVITY WITH CANTILEVER THICKNESS FOR UNTENSIONED DIAPHRAGM (SINGLE CANTILEVER TYPE PICKUP) — CASE A.

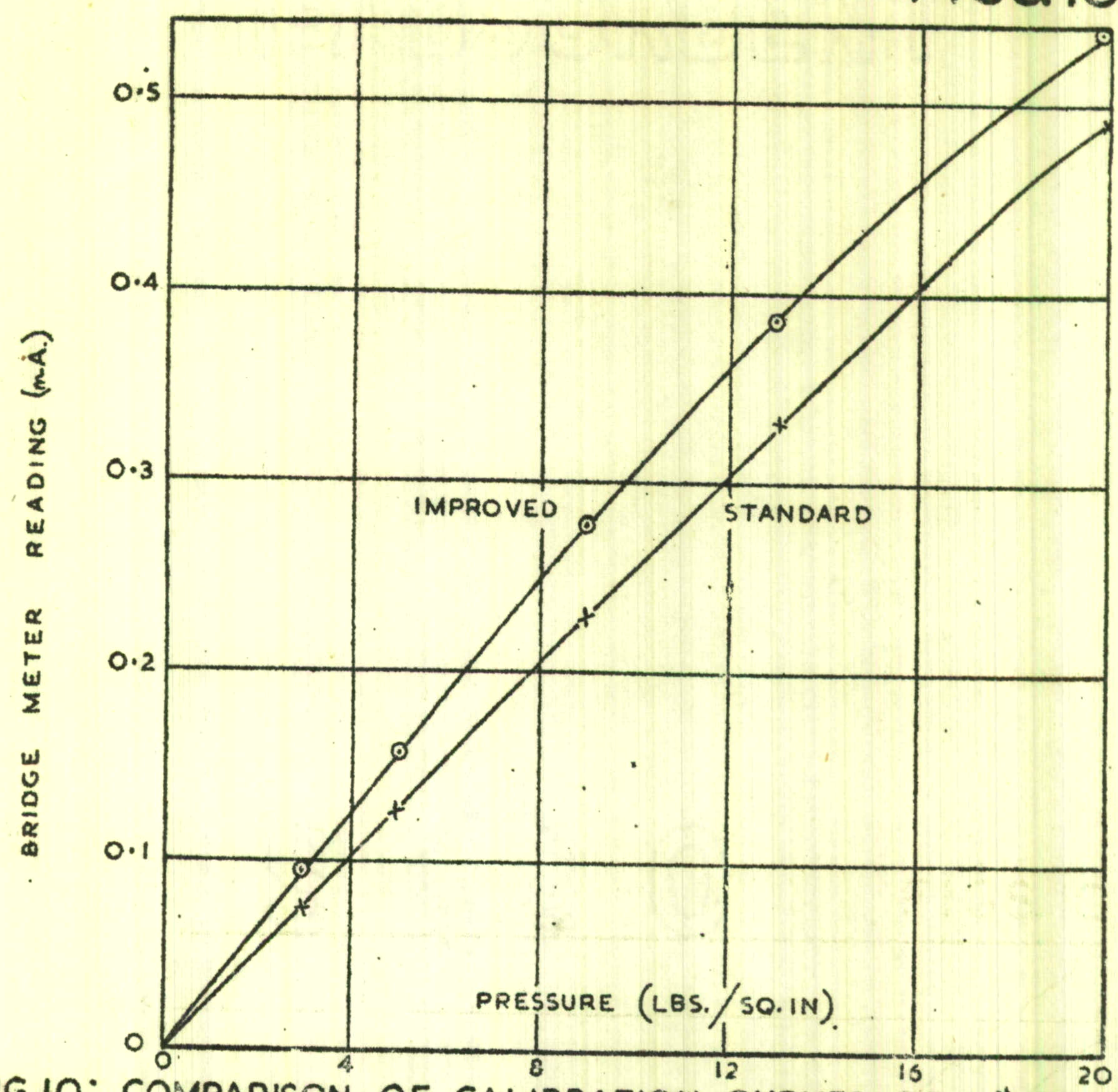


FIG. 10: COMPARISON OF CALIBRATION CURVES OF A "STANDARD" PICKUP AND A SIMILAR PICKUP AT OPTIMUM CANTILEVER THICKNESS FOR CASE A.

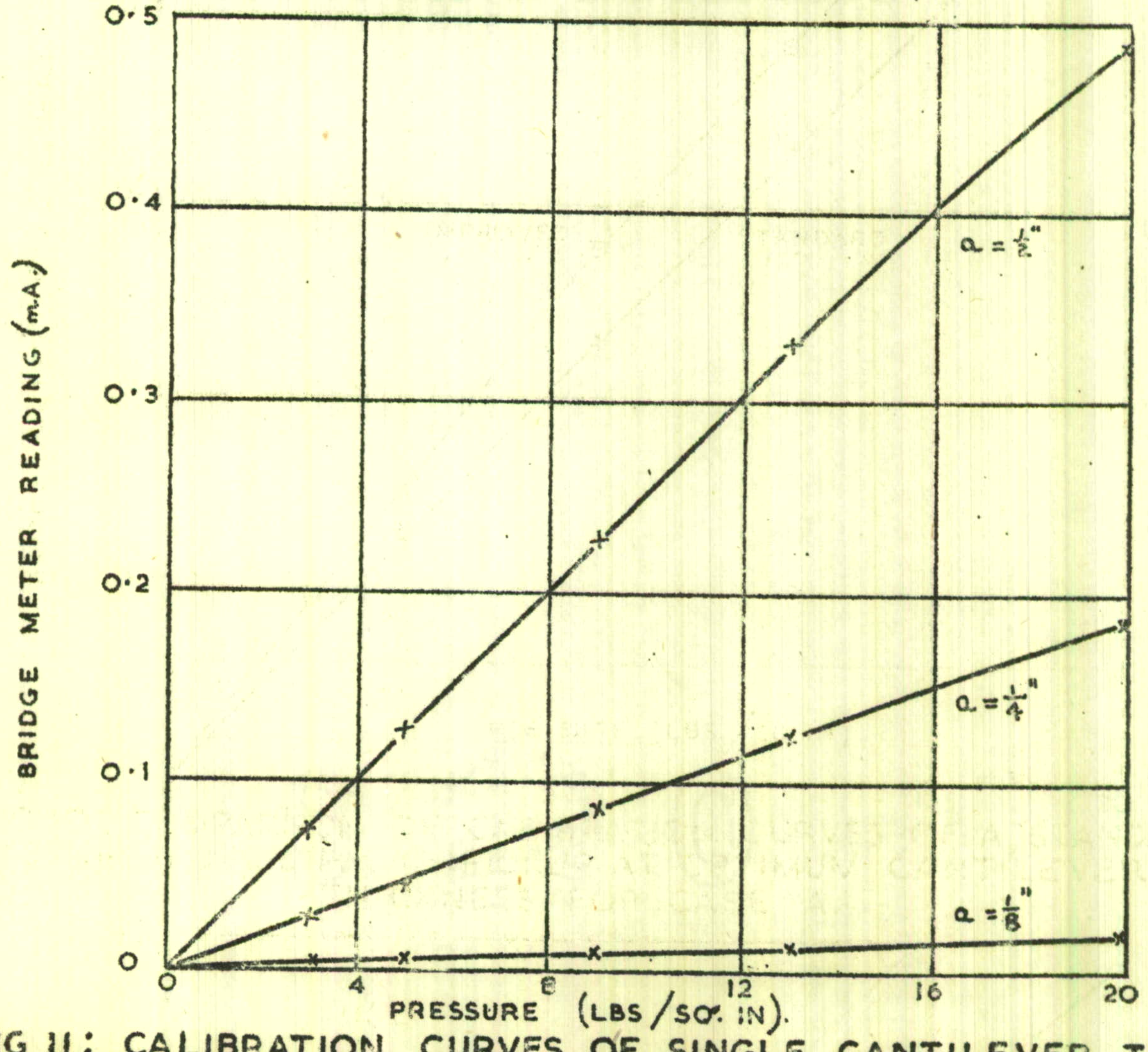
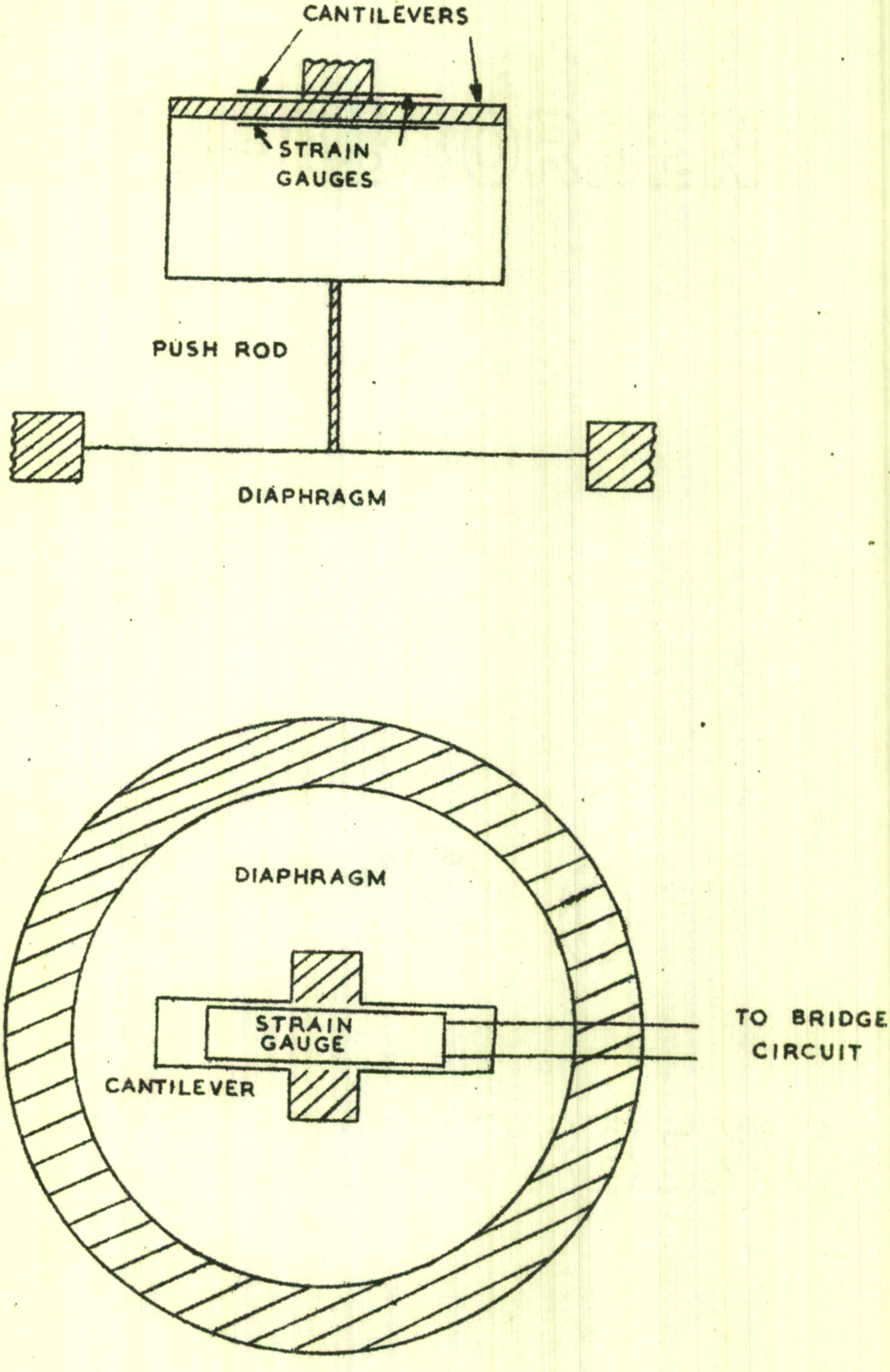


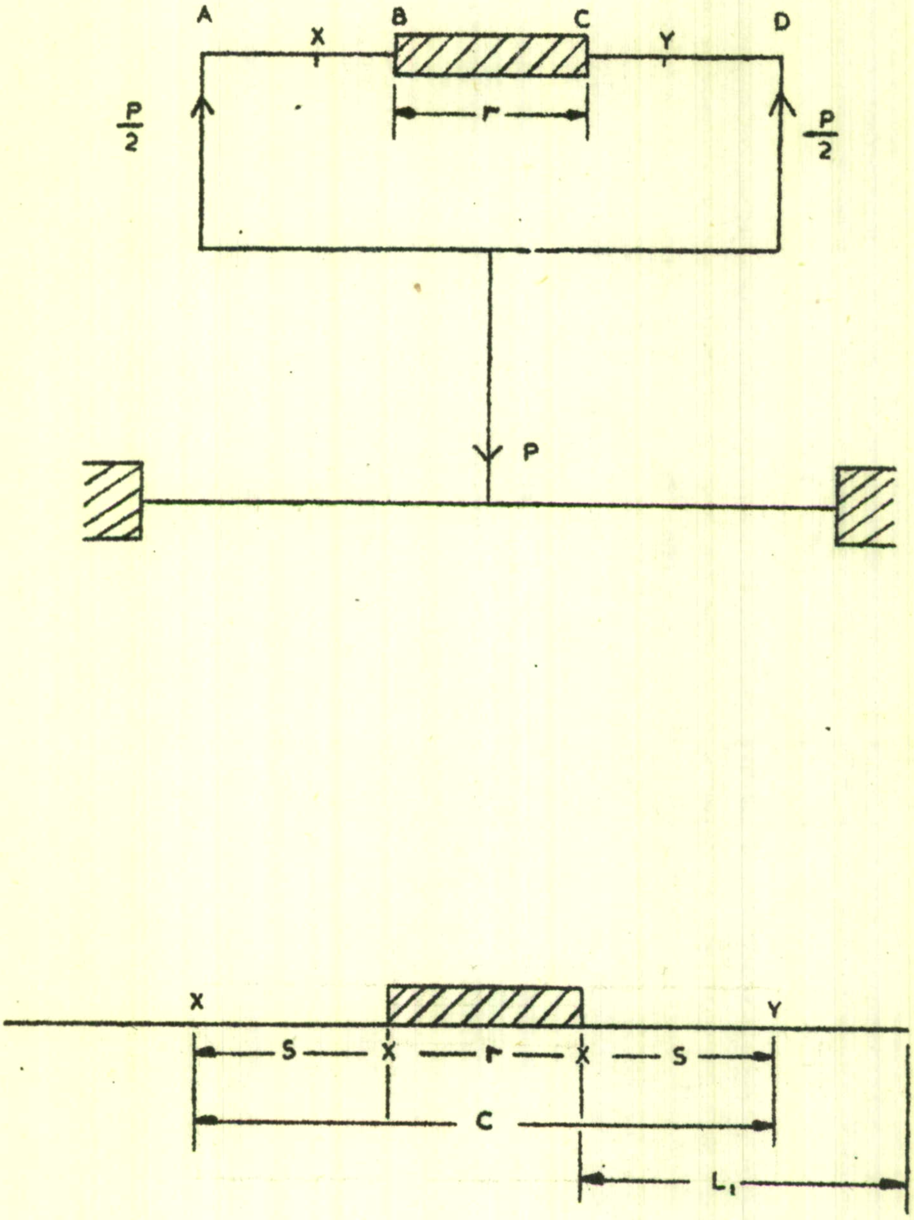
FIG 11: CALIBRATION CURVES OF SINGLE CANTILEVER TYPE PICKUPS OF VARIOUS DIAPHRAGM RADII AT OPTIMUM CANTILEVER THICKNESSES FOR CASE A.

FIG.12.



SCHEMATIC ARRANGEMENT OF TWIN CANTILEVER TYPE PICKUP.

FIG. 13.



NOTATION USED IN SECTION 3.

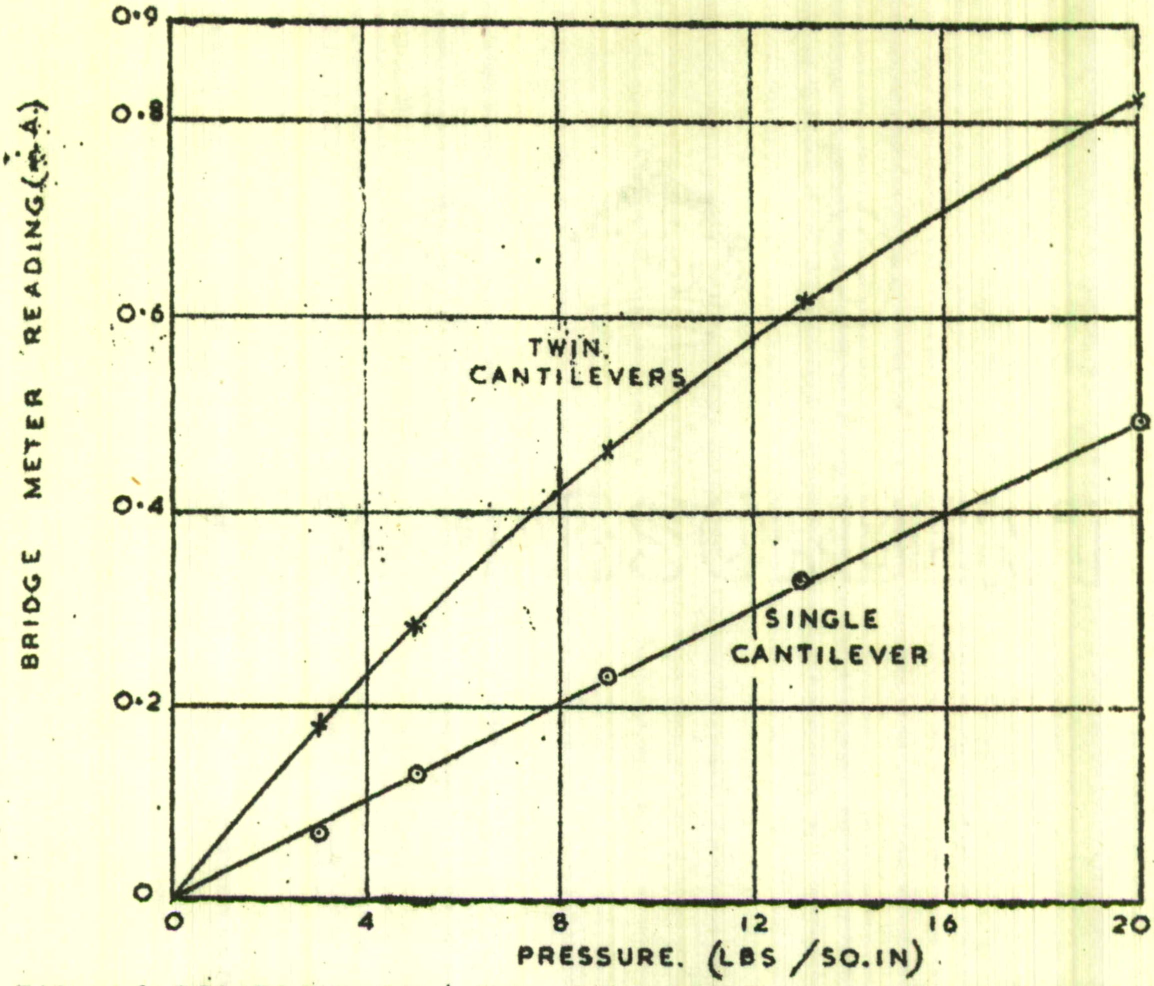


FIG 14: COMPARISON OF CALIBRATION CURVES OF SINGLE AND TWIN CANTILEVER TYPE PICKUPS, BOTH AT OPTIMUM CANTILEVER THICKNESSES.

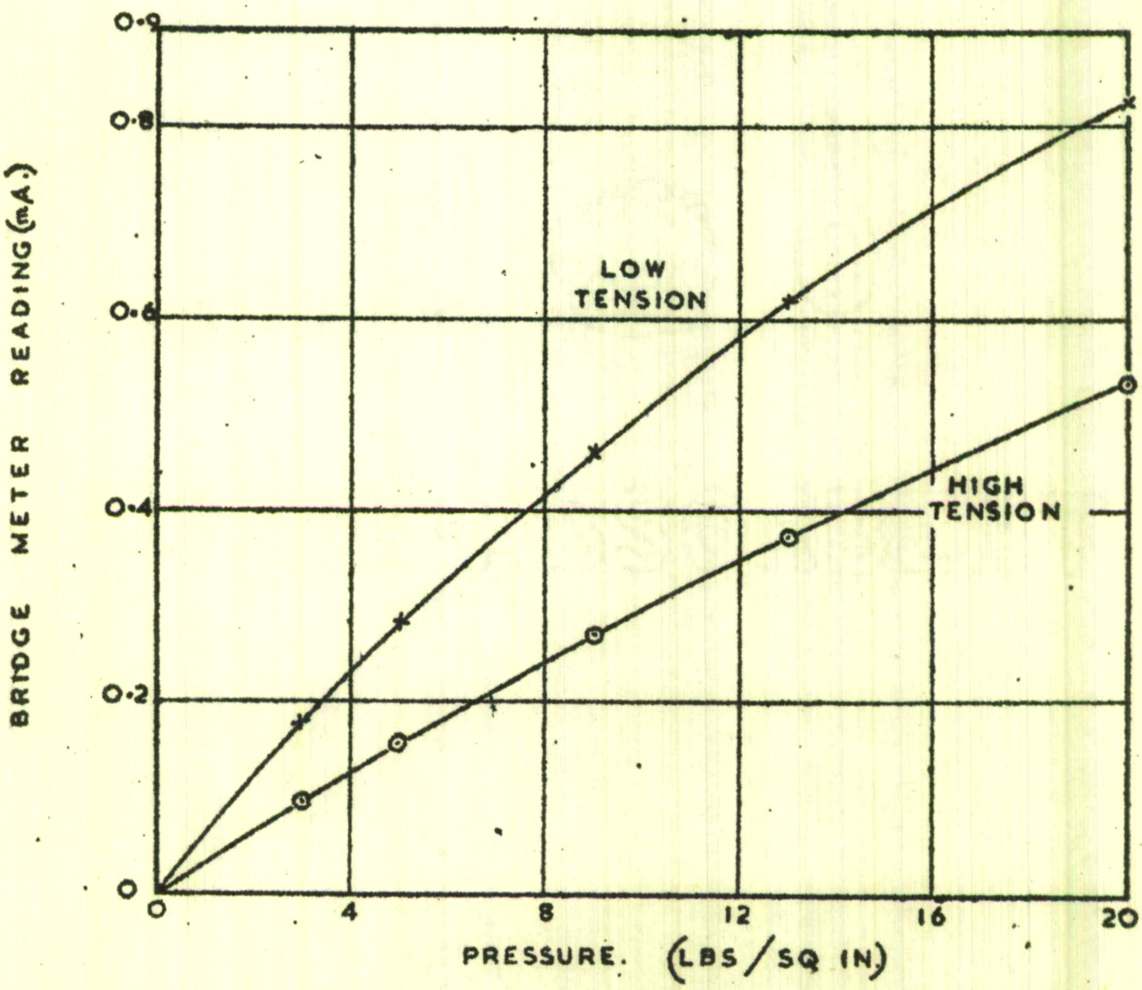


FIG 15: COMPARISON OF CALIBRATION CURVES OF TWIN CANTILEVER TYPE PICKUPS WITH DIFFERENT DIAPHRAGM TENSIONS.

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